combiner, are used to establish this condition and to compensate for variations in the frequency response of the laser and receiver. This equalisation provides a significant reduction in IMD, given the resonant response of REC2. Since the receiver sensitivity is highest for channels near the centre of the octave, the OMD for these channels can be reduced. Typically, for the centre channels, OMD $\simeq 0{\cdot}056$, and for the outer channels OMD $\simeq 0{\cdot}12$. With 4 dBm optical power into the fibre at the head-end and a total system loss of 21 dB, each of the eight channels could be received with $P_e < 10^{-9}$. The IMD penalty, measured by comparing P_e for the test channel, with and without the other seven channels, was negligible. Also, injection of light from one laser into the other does not introduce any measurable penalty. Optical isolation is not required.

In summary, we have demonstrated one example of a bidirectional SCM-based distribution system. Penalties associated with intermodulation distortion, closely-spaced channels, optical reflections and bidirectional transmission are all negligible. A variety of related architectures are feasible, making SCM an attractive technique for multiuser lightwave networks.

Acknowledgment: This letter was presented as a postdeadline paper at the Optical Fiber Conference OFC '88, New Orleans, Jan. 1988.

T. E. DARCIE
P. P. IANNONE
B. L. KASPER
J. R. TALMAN
C. A. BURRUS
T. A. BAKER

AT&T Bell Laboratories
Crawford Hill Laboratory
Holmdel, NJ 07733, USA

26th January 1988

References

1 DARCIE, T. E.: 'Subcarrier multiplexing for multiple-access lightwave networks', *J. Lightwave Technol.*, 1987, **LT-5**, pp. 1103–1110
2 DARCIE, T. E., DIXON, M. E., KASPER, B. L., and BURRUS, C. A.: 'Lightwave system using microwave subcarrier multiplexing', *Electron. Lett.*, 1986, **22**, pp. 774–775
3 BOWERS, J. E.: 'Microwave signal transmission with high-speed GaInAsP lasers', Optical fiber communications conference (OFC/IOOC '87), Reno, NV, January 19–22, 1987, WK1, p. 175
4 WAY, W. I., KRAIN, W., and WOLFF, R. S.: 'A 35 km satellite earth station 1·3 μm fiber optic microwave multicarrier transmission system', *ibid.*, 1987, PDP18, pp. 74–77
5 OLSHANSKY, R., and LANZISERA, V.: '60-channel FM video subcarrier multiplexed optical communication system', *Electron. Lett.*, 1987, **23**, pp. 1196–1198
6 DARCIE, T. E., TUCKER, R. S., and SULLIVAN, G. J.: 'Intermodulation and harmonic distortion in GaInAsP lasers', *ibid.*, 1985, **21**, pp. 665–666
7 IANNONE, P., and DARCIE, T. E.: 'Multichannel intermodulation distortion in high-speed GaInAsP lasers', *ibid.*, 1987, **23**, pp. 1361–1362
8 DARCIE, T. E., KASPER, B. L., TALMAN, J. R., and BURRUS, C. A.: 'Resonant PIN-FET receivers for lightwave subcarrier systems', accepted for publication in *J. Lightwave Technol.*

CONSTRUCTING NEW PERFECT BINARY ARRAYS

Indexing terms: Information theory, Binary sequences, Digital systems

Only a small number of different sizes are known for which there exist two-dimensional perfect binary arrays. Construction methods are given which generate new two-dimensional perfect binary arrays, four of which are larger than any previously reported.

Introduction: A perfect binary array is an n-dimensional array, all of whose entries are $+1$ or -1, with the property that all its out-of-phase periodic autocorrelation coefficients are zero. Such arrays were initially studied by Calabro and Wolf in 1968[1] and subsequent results on the existence of these arrays have been achieved in 1979 by Chan *et al.*[2] and in 1987 by Bömer and Antweiler.[3] We are particularly concerned here with two-dimensional perfect binary arrays, potential applications for which include 2D synchronisation, image coding and data compression.[4]

More formally, suppose

$$A = (a_{ij}), \quad 0 \le i \le s-1, \quad 0 \le j \le t-1,$$

is an $s \times t$ array of '$+1$'s and '-1's. As throughout, this refers to an array with s rows and t columns. Define the periodic autocorrelation function at displacement (u, v), or PACF(u, v) by

$$\text{PACF}(u, v) = \sum_{i=0}^{s-1} \sum_{j=0}^{t-1} a_{ij} \cdot a_{(i+u)(j+v)}$$

where $0 \le u \le s-1$, $0 \le v \le t-1$ and $i+u$, $j+v$ are reduced modulo s, t as necessary.

Then A is said to be a perfect binary array, or a PBA(s, t), if PACF$(u, v) = 0$ for all (u, v) except $(u, v) = (0, 0)$.

If $s = t = 1$ then A is said to be trivial. The following example of a PBA(6, 6) is due to Bömer and Antweiler:[3]

−	+	+	+	+	−
+	−	+	+	+	−
+	+	−	+	+	−
+	+	+	−	+	−
+	+	+	+	−	−
−	−	−	−	−	+

Calabro and Wolf[1] proved the following:

Theorem 1: If A is a nontrivial PBA(s, t) then

(i) $s \cdot t = 4k^2$ for some integer k.

(ii) A must contain either $2k^2 + k$ or $2k^2 - k$ entries of $+1$.

It is also clear that, if A is a PBA(s, t) then

(i) $-A$ (i.e. the array derived from A by changing the signs of all the elements in A) is also a PBA(s, t).

(ii) An array derived from A by any combination of cyclic rotation of rows, cyclic rotation of columns, reversal of order of rows and reversal of order of columns, is also a PBA(s, t).

(iii) A^t (the transpose of A) is a PBA(t, s).

(iv) If A has an even number of rows (or columns) then changing the sign of all the elements in the odd rows (or columns) of A gives a PBA(s, t).

There are only 10 pairs of values (s, t) for which a nontrivial PBA(s, t) is known to exist, as follows: (1, 4), (2, 2), (4, 4),[1] (3, 12), (6, 6), (12, 12),[2] (2, 8),[5] (6, 24),* (4, 16) and (8, 8).† The (unique) PBA(1, 4) is the only known nontrivial one-dimensional PBA. In fact it is conjectured that no others exist.[6]

We now exhibit some construction methods which enable the construction of PBAs of four new sizes, namely (8, 32), (16, 16), (12, 48) and (24, 24). These methods of construction also hold out the hope of constructing further new arrays.

Before proceeding we need a further definition. Suppose $A = (a_{ij})$ is an $s \times t$ array of '$+1$'s and '-1's, with the property that t is even, $t = 2T$, say. Then define the partial

* LÜKE, H. D., and BÖMER, L.: 'Perfect binary arrays', in preparation
† BÖMER, L., and ANTWEILER, M.: 'Two-dimensional perfect binary arrays with 64 elements', in preparation

periodic autocorrelation function at displacement (u, v), or PPACF(u, v), by

$$\mathrm{PPACF}(u, v) = \sum_{i=0}^{s-1} \sum_{j=0}^{T-1} a_{i(2j)} \cdot a_{(i+u)(2j+v)}$$

where $0 \le u \le s-1$, $0 \le v \le t-1$, and $i+u$, $2j+v$ are reduced modulo s, t as necessary.

Construction method 1: Suppose $A = (a_{ij})$ is a PBA$(s, 2t)$. Moreover, suppose $B = (b_{ij})$ is the $2s \times t$ array defined by

$$b_{(2i+d)j} = a_{i(2j+d)}$$

where $0 \le i < s$, $0 \le j < t$ and $0 \le d \le 1$.

Then B is a PBA$(2s, t)$ if and only if A has partial periodic autocorrelation function satisfying

$$\mathrm{PPACF}(u, 2v+1) - \mathrm{PPACF}(u+1, 2v-1) = 0$$

for every u, v satisfying $0 \le u < s$, $0 \le v < t$, and where $u+1$ is reduced modulo s and $2v+1$ and $2v-1$ are reduced modulo $2t$ as necessary.

As an example consider the following PBA(2, 8):

+	+	+	+	+	+	−	−
+	−	+	−	+	−	−	+

This array satisfies PPACF$(0, v)$ = PPACF$(1, v) = 0$ for all odd v, where $0 < v < 8$, and hence can be used to yield a PBA(4, 4), namely

+	+	+	−
+	+	+	−
+	+	+	−
−	−	−	+

Finally, note that a very similar construction is used in Reference 3.

Construction method 2: Suppose

$$B = (b_{ij}), \quad 0 \le i \le s-1, \quad 0 \le j \le t-1,$$

is an $s \times t$ array of '+1's and '−1's. Define the periodic quasi-autocorrelation function at displacement (u, v), or QACF(u, v) by

$$\mathrm{QACF}(u, v) = \sum_{i=0}^{s-u-1} \sum_{j=0}^{t-1} b_{ij} \cdot b_{(i+u)(j+v)} - \sum_{i=s-u}^{s-1} \sum_{j=0}^{t-1} b_{ij} \cdot b_{(i+u)(j+v)}$$

where $0 \le u \le s-1$, $0 \le v \le t-1$, and $i+u$, $j+v$ are reduced modulo s, t as necessary.

Then B is said to be a quasiperfect binary array, or a QPBA(s, t), if QACF$(u, v) = 0$ for all (u, v) except $(u, v) = (0, 0)$.

The following are examples of QPBAs:

+		+	+		+	+	+	+		+	−	−	+	−	+
+		+	−		+	+	−	−		−	−	−	+	−	+
					+	−	+	−		+	+	−	−	+	−
					+	+	−	−		−	−	−	−	−	+
										−	−	−	+	+	+
										+	+	+	−	+	+

Now, suppose A is a PBA(s, t) and B is a QPBA(s, t). Let D be the $2s \times t$ array obtained by joining two copies of A together one on top of the other, and let E be the $2s \times t$ array obtained by similarly joining a copy of B to a copy of $-B$. Let F be the $2s \times 2t$ array obtained by interleaving alternate columns of D and E. Then F is always a PBA$(2s, 2t)$. It is straightforward to check that any F constructed this way has the property that PPACF$(u, 2v+1) = 0$ for all u, v $(0 \le u < 2s, 0 \le v < t)$ and so construction method 1 can be applied to F. Hence:

Theorem 2: If there exists a PBA(s, t) and a QPBA(s, t) then there exists a PBA$(2s, 2t)$ and a PBA$(4s, t)$.

Construction method 3: Suppose

$$C = (c_{ij}), \quad 0 \le i \le s-1, \quad 0 \le j \le t-1,$$

is an $s \times t$ array of '+1's and '−1's. Define the periodic doubly-quasi-autocorrelation function at displacement (u, v), or DQACF(u, v) by

$$\begin{aligned}\mathrm{DQACF}(u, v) = &\sum_{i=0}^{s-u-1} \sum_{j=0}^{t-v-1} b_{ij} \cdot b_{(i+u)(j+v)} - \sum_{i=s-u}^{s-1} \sum_{j=0}^{t-v-1} b_{ij} \cdot b_{(i+u)(j+v)} \\ &- \sum_{i=0}^{s-u-1} \sum_{j=t-v}^{t-1} b_{ij} \cdot b_{(i+u)(j+v)} + \sum_{i=s-u}^{s-1} \sum_{j=t-v}^{t-1} b_{ij} \cdot b_{(i+u)(j+v)}\end{aligned}$$

where $0 \le u \le s-1$, $0 \le v \le t-1$, and $i+u$, $j+v$ are reduced modulo s, t as necessary.

Then C is said to be a doubly-quasi-perfect binary array, or a DQPBA(s, t), if DQACF$(u, v) = 0$ for all (u, v) except $(u, v) = (0, 0)$.

The following are examples of DQPBAs:

+	+		+	+	+	+		−	+	−	−	+	+
+	+		+	+	+	+		+	−	+	−	+	+
			+	−	−	+		−	+	−	−	−	+
			−	+	+	−		−	−	−	+	−	+
								+	+	−	−	−	+
								+	+	+	+	+	+

Now suppose B is a QPBA(s, t) and C is a DQPBA(s, t). Let D be the $s \times 2t$ array obtained by joining two copies of B together horizontally, and let E be the $s \times 2t$ array obtained by similarly joining a copy of C to a copy of $-C$. Finally let F be the $2s \times 2t$ array obtained by interleaving alternate rows of D and E. Then F is always a QPBA$(2s, 2t)$.

Theorem 3: If there exists a QPBA(s, t) and a DQPBA(s, t) then there exists a QPBA$(2s, 2t)$.

New 2-dimensional perfect binary arrays: Since we know that a DQPBA(4, 4) and a QPBA(4, 4) exist, Theorem 3 gives a QPBA(8, 8). Additionally, a QPBA(4, 4) and a PBA(4, 4) can be used to give a PBA(8, 8) (by Theorem 2). Using Theorem 2 again, we obtain the desired PBA(16, 16) and PBA(32, 8). In exactly the same way, a PBA(24, 24) and a PBA(48, 12) may be derived from a PBA(6, 6), a QPBA(6, 6) and a DQPBA(6, 6), examples of which are given above.

Concluding remarks: The above construction methods give strong clues about how even larger PBAs might be constructed. In particular, since a QPBA(8, 8) exists, if a DQPBA(8, 8) could be found, then it would be possible to construct PBAs having 1024 elements.

Acknowledgments: We would like to acknowledge the generosity of Prof. Lüke, Mr. Bömer and Mr. Antweiler in providing us with prepublication copies of their work. Their examples of a PBA(8, 8) and a PBA(4, 16) have provided

inspiration for much of our work. We would also like to thank Dr. P. Wild and Prof. F. Piper for many stimulating conversations and suggestions.

J. JEDWAB
C. MITCHELL

15th February 1988

Hewlett–Packard Laboratories
Filton Road
Stoke Gifford
Bristol BS12 6QZ, United Kingdom

References

1 CALABRO, D., and WOLF, J. K.: 'On the synthesis of two-dimensional arrays with desirable correlation properties', *Inf. & Control*, 1968, **11**, pp. 537–560
2 CHAN, Y. K., SIU, M. K., and TONG, P.: 'Two-dimensional binary arrays with good autocorrelation', *Inf. & Control*, 1979, **42**, pp. 125–130
3 BÖMER, L., and ANTWEILER, M.: 'Perfect binary arrays with 36 elements', *Electron. Lett.*, 1987, **23**, pp. 730–732
4 GREEN, D. H.: 'Structural properties of pseudorandom arrays and volumes and their related sequences', *IEE Proc. E, Comput. & Digit. Tech.*, 1985, **132**, pp. 133–145
5 LÜKE, H. D.: 'Zweidimensionale folgen mit perfekten periodischen korrelationsfunktionen, *Frequenz*, 1987, **41**, pp. 131–137
6 BAUMERT, L. D.: 'Cyclic difference sets', (Springer-Verlag, Berlin, 1971)

RESONANTLY ENHANCED TWO-WAVE MIXING VIA POPULATION PULSATIONS IN SEMICONDUCTOR LASER AMPLIFIERS

Indexing terms: Semiconductor lasers, Optical properties of substances, Multiplexing, Mixers

We demonstrate theoretically the existence of enhanced probe gain via population pulsations in nearly degenerate two-wave mixing in a semiconductor amplifier. The enhancement arises from the injection current modulation frequency coinciding with the pump/probe detuning. Probe saturation is shown to occur in less than 10 μm cavity length.

Interband population pulsations in semiconductor lasers have been studied recently[1] as a mechanism for driving nearly degenerate four-wave mixing (NDFWM). Such studies indicate that significant harmonic mixing occurs for four nearly degenerate beams passing through a semiconductor laser when operated below threshold as a travelling-wave amplifier.[1] The interaction of various frequencies in travelling-wave amplifiers is of practical importance in optical communications as intermodulation effects will limit the multiplexing capability available.[2]

In this letter we analyse the simpler but closely related process of two-wave mixing in semiconductor laser amplifiers. Two-wave mixing via the selfdiffraction of two grating-forming beams is well known in photorefractive materials[2] and has been shown in that context to yield useful operations, such as image amplification and edge enhancement.[3] The amplifier proposed here may have similar applications. We demonstrate that amplification is enhanced by several orders of magnitude by modulating the device at a frequency resonant with the detuning frequency.

The geometry is shown in Fig. 1. A strong pump wave E_{ω_0} and a weak probe E_{ω_1} interact within the semiconductor medium to produce an interband population pulsation at the beat frequency $\Omega = \omega_0 - \omega_1$. This is resonantly driven by a modulated current, $I(t) = I_0[1 + m\cos(\Omega t)]$, where $I_0(1 + m) < I_{th}$. Above-threshold operation is more complicated due to the additional beams internally generated via stimulated emission.

The coupled wave equations describing pump/probe energy exchange are obtained via a modification of the four-wave mixing equations in Reference 1:

$$\frac{dA_0}{dz} + \alpha_0 A_0 = 0 \quad (1)$$

$$\frac{dA_1}{dz} + \alpha_1 A_1 = \gamma A_0 \quad (2)$$

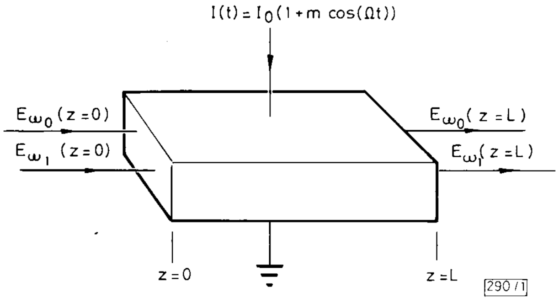

Fig. 1 *Geometry for enhanced two-wave mixing in semiconductor laser amplifiers*

The amplitudes A_0 and A_1 are assumed slowly varying and the coupling constant is given by

$$\gamma = \frac{\Gamma}{2}(1 - i\beta)\frac{I_0 g m \tau_e}{qV}\frac{1}{1 + P_0 - i\Omega T_e} \quad (3)$$

where g is the gain constant, τ_e is the carrier lifetime, q is the electronic charge, V is the device active volume, P_0 is the power inside the active volume normalised to the saturation power $P_s = \hbar\omega/\Gamma_g \tau_p$,[1] β is the linewidth enhancement factor and Γ is the mode confinement factor. The absorption constants α_0 and α_1 are defined in Reference 1. The analytic solution of eqns. 1 and 2 yields an expression for the intensity gain seen by the probe, $I_1(L)/I_1(0)$ and is given by

$$\frac{I_1(L)}{I_1(0)} = \left|\frac{\gamma}{\alpha_1 - \alpha_0}\right|^2 |\kappa|^2 + 2\left|\frac{\gamma}{\alpha_1 - \alpha_0}\right| \mathrm{Re}\,[\kappa \exp(-\alpha_1^* z)] + \exp[-2\mathrm{Re}\,(\alpha_1 z)] \quad (4)$$

where

$$\kappa = \left(\frac{I_0(0)}{I_1(0)}\right)^{1/2} (\exp(-\alpha_0 z) - \exp(-\alpha_1 z)) \quad (5)$$

Fig. 2 shows the intensity gain as a function of interaction length for various modulation depths. Device parameters from Reference 1 were used which are typical for GaInAsP lasers. It is seen that for an incident intensity ratio $I_{pump}/I_{probe} = 50$,

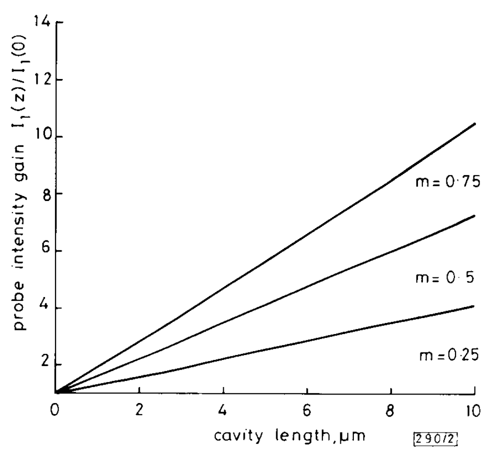

Fig. 2 $I_1(z)/I_1(0)$ *against z for various modulation depths*
Pump/probe ratio, $I_0(0)/I_1(0) = 50$, $P_0 = 0{\cdot}1$